Wayne
Township Public Schools

**Mathematics**

**Grade 12**

**Calculus Honors**

Dr. Mark Toback, Superintendent

Committee: Todd Green

Compliance Update Completed on June 2022

*This curriculum may be modified through varying techniques, strategies, and materials as per an individual student's Individualized Educational Plan (IEP)*

# Approved by the Wayne Township Board of Education at the regular meeting held on November 15, 2018.

**New Jersey Student Learning Standards For Mathematics**

**Standards for Mathematical Practice**

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning , strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students:

- explain to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution attempt.
- consider analogous problems, and try special cases and simpler forms of the original problem.
- monitor and evaluate their progress and change course if necessary.
- transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- explain correspondences between equations, verbal descriptions, tables, and graphs.
- draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- use concrete objects or pictures to help conceptualize and solve a problem.
- check their answers to problems using a different method.
- ask themselves, "Does this make sense?"
- understand the approaches of others to solving complex problems.

**2 Reason abstractly and quantitatively.**

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.

- ✓ *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
- ✓ *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
- use  quantitative reasoning that entails creating a coherent representation of quantities, not just how to compute them
- know and flexibly use different properties of operations and objects.

**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context
- compare the effectiveness of plausible arguments
- distinguish correct logic or reasoning from that which is flawed
- ✓ elementary students construct arguments using objects, drawings, diagrams, and actions..
- ✓ later students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions

**4 Model with mathematics.**

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- ✓ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
- ✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- simplify a complicated situation, realizing that these may need revision later.

- identify important quantities in a practical situation
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5 Use appropriate tools strategically.**

Mathematically proficient students:

- consider available tools when solving a mathematical problem.
- are familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

**6 Attend to precision.**

Mathematically proficient students:

- try to communicate precisely to others.
- use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.

✓ In the elementary grades, students give carefully formulated explanations to each other.

✓ In high school, students have learned to examine claims and make explicit use of definitions.

**7 Look for and make use of structure.**

Mathematically proficient students:

- look closely to discern a pattern or structure.

✓ Young students might notice that three and seven more is the same amount as seven and three more.

✓ Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.

✓ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7.

- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students:

- notice if calculations are repeated
- look both for general methods and for shortcuts.
- maintain oversight of the process, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

*Wayne School District*
**Curriculum Format**

| | |
|---|---|
| **Content Area/ Grade Level/ Course:** | **Mathematics**<br>**12**<br>**Calculus Honors** |
| **Unit Plan Title:** | **Functions, Limits & Continuity** |
| **Time Frame** | **30 days** |

**Anchor Standards/Domain***   ***i.e: ELA: reading, writing  i.e.:  Math: Algebra**

**N-RN: Number and Quantity – The Real Number System**
**N-Q: Number and Quantity – Quantity**
**A-SSE: Algebra – Seeing Structure in Expressions**
**A-APR: Algebra – Arithmetic with Polynomials and Rational Expressions**
**A-CED: Algebra – Creating Equations**
**A-REI: Algebra- Reasoning with Equations and Inequalities**
**F-IF:  Functions – Interpreting Functions**
**F-BF: Functions – Building Functions**
**F-TF: Functions - Trigonometric Functions**

**Unit Overview**

Calculus Honors is primarily concerned with developing the students' understanding of the concepts of Calculus and providing experience with its methods and applications. The course emphasizes a multi-representational approach to Calculus, with concepts, results, and problems being expressed numerically, analytically, geometrically, and verbally. Technology reinforces the relationships among the multiple representations of functions, to confirm written  work, to implement experimentation, and to assist in interpreting results.

In calculus, many concepts are developed by first considering a discrete model and then examining the limiting case. This makes the idea of limits essential for developing important ideas in Calculus.  Through the use limits and continuity, the unit will cover approximation, theorems/rules (Sandwich, Intermediate), asymptotes, and local/global values of a function.Students must have an understanding of limits and be able to compute limits graphically, algebraically, and numerically with the help of a graphing calculator.  They should be able to examine one sided limits and limits involving infinity.  Students should understand the algebraic procedures for finding limits with indeterminate forms.  They should also be able to apply the definition of a limit to determine continuity of a function.  Applications of limits will involve calculating average rates of change and instantaneous rates of change.  This will set the foundation for finding the limit definition of the derivative function.

**Standard Number(s)**   *** i.e:   Math: F.LE.A.4       i.e.: NJSLSA.R4.**

N-RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*

N-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-RN.B.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

N-Q.A.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays

A-SSE.A.1. Interpret expressions that represent a quantity in terms of its context

A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it

A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-BF.A.1 Write a function that describes a relationship between two quantities.

F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF.B.4 Find inverse functions.

F-BF.B.5 Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

F-TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.A.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for πx, π+x, and 2π–x in terms of their values for x, where x is any real number.

F-TF.A.4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

K-12.MP.1 Make sense of problems and persevere in solving them.

K-12.MP.2 Reason abstractly and quantitatively.

K-12.MP.3 Construct viable arguments and critique the reasoning of others.

K-12.MP.4 Model with mathematics

K-12.MP.5 Use appropriate tools strategically.

K-12.MP.6 Attend to precision

K-12.MP.7 Look for and make use of structure.

K-12.MP.8 Look for and express regularity in repeated reasoning.

**Intended Outcomes - {Essential Questions}**

- What is a limit?
- In what real-world situations would the calculation of a limit be useful?
- What is continuity?
- What are vertical and horizontal aymptotes?

**Enduring Understandings**

- Analysis of the critical elements of functions is essential to calculus.
- Functions can be analyzed graphically by their limiting behavior and rates of change.
- Functions can be analyzed using their table of values.
- Technology can be used at various stages to enhance understanding using its power to visualize and computate
- Functions approach values at local and global values
- The relationship between limits and how they describe the behavior of a function

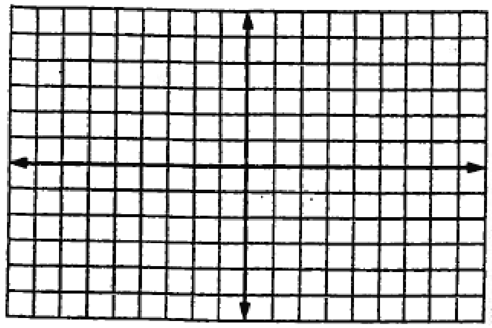
**In this unit plan, the following 21st Century themes and skills are addressed.**

| *Check all that apply.*<br>**21st Century Themes** | | *Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*<br>**21st Century Skills** | |
|---|---|---|---|
| | **Global Awareness** | X | **Creativity and Innovation** |
| | **Environmental Literacy** | X | **Critical Thinking and Problem Solving** |
| | **Health Literacy** | X | **Communication** |
| | **Civic Literacy** | X | **Collaboration** |
| X | **Financial, Economic, Business, and Entrepreneurial Literacy** | | |

**Student Learning Targets/Objectives (Students will know/Students will understand)**

- write an equation and sketch a graph of a line given specific information
- recognize the domain and range of a function and even or odd functions
- interpret and find formulas for piecewise functions and compositions of functions
- identify one-to-one functions
- identify the properties and characteristics of exponential and logarithmic functions and trigonometric functions
- calculate average and instantaneous rates of change (speeds)
- Calculate limits intuitively for both one- and two-sided limits
- Define and calculate limit of a function, if it exists, and apply the properties of limits
- Use the Sandwich Theorem to find certain limits indirectly
- Find and verify end behavior models for various functions
- Calculate limits as $x \to \pm\infty$ and identify horizontal asymptotes
- Use limits of $\pm\infty$ to locate vertical asymptotes
- Understand the meaning of a continuous function, remove removable discontinuities by extending or modifying a function
- Apply the Intermediate Value Theorem and the properties of algebraic combinations and compositions of continuous functions

Sketch a function that meets the conditions indicated. Identify all asymptotes.



1. $\lim_{x\to-\infty} f(x) = 2$ $\quad \lim_{x\to1} f(x) = 4$ $\quad \lim_{x\to3^-} f(x) = -\infty$

$\lim_{x\to3^+} f(x) = -\infty$ $\quad \lim_{x\to\infty} f(x) = \infty$

If $f(x) = \begin{cases} k^2 - x^2, & x \le 2 \\ 1.5kx, & x > 2 \end{cases}$, find the value of the constant $k$ so that $\lim_{x\to2} f(x)$ exists.

The functions $f$ and $g$ are continuous for all real numbers. The table below gives values of the functions at selected values of $x$. The function $h$ is given by $h(x) = f\left(g(x)\right) - 6$. Explain why there must be a value $r$ for $1 < r < 3$ such that $h(r) = -5$.

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | 6 | 9 | 10 | $-1$ |
| $g(x)$ | 2 | 3 | 4 | 5 |

**Assessments (Pre, Formative, Summative, Other)** ***Denote required common assessments with an ****

Quizzes
Homework
Unit Test
Sample AP Multiple Choice problems
Warm up problems

**Teaching and Learning Activities**

| *Activities* | The course lends itself to a mode of instruction that engages the students in a multi-representational approach to studying calculus concepts. Many "what if' questions provide great insights or a basis for discussion and help to validate understanding at various levels of development as each unit is explored. While problem sets and explorations may be rigorous and challenging, they are designed so that the students can be successful if they utilize those resources available to them such as classroom discussions, group explorations, projects, and peer and instructor interactions. Instructional modes attempt to address the variety of learning styles that include visual learners, auditory learners, and kinesthetic learners. |
|---|---|

| | |
|---|---|
| | Students link arms kinestetic activity to discuss notion of continuity and Intermediate Value Theorem. |
| *Differentiation Strategies* | Alternative assessment projects to demonstrate mastery<br>Peer to Peer Tutoring<br>Implementation of Visual Representations- Calculus in Motion<br>Technology Implementation<br>Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of limits<br>Allow students to work in small groups<br>Provide access to Khan Academy Videos<br>Provide opportunities for questions<br>Differentiation Strategies for Special Education Students<br>Differentiation Strategies for Gifted and Talented Students<br>Differentiation Strategies for ELL Students<br>Differentiation Strategies for At Risk Students |
| *Honors* | Completing AP free-response questions developed by College Board |

**Resources**

www.collegeboard.com: provides information about AP exams

http://archives.math.utk.edu/visual.calculus/: an extremely thorough site for pre-calculus through integral calculus with accompanying tutorials, practice problems, interactive quizzes and animations

http://online.math.uh.edu/HoustonACT/: contains Powerpoint presentations

www.calculus-help.com/funstuff/phobe.html: animated explanations of the first two chapters of calculus

www.calculusabc.com: teacher and student resources containing multiple choice problems for each unit and well as a forum for teachers to exchange teaching ideas

http://clem.mscd.edu/~talmanl/APCalculus.html: FRQ solutions, explanations, links to number of other sites and much more

http://www.calculus.org/

http://www.calculus-help.com/

AP Central

Khan Academy

Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.

| | |
|---|---|
| **Content Area/ Grade Level/ Course:** | **Mathematics**<br>**12**<br>**Calculus Honors** |
| **Unit Plan Title:** | **Derivatives** |
| **Time Frame** | **40 days** |

**Anchor Standards/Domain*     *i.e: ELA: reading, writing  i.e.:  Math: Algebra**

**N-RN: Number and Quantity – The Real Number System**
**N-Q: Number and Quantity – Quantity**
**A-SSE: Algebra – Seeing Structure in Expressions**
**A-APR: Algebra – Arithmetic with Polynomials and Rational Expressions**
**A-CED: Algebra – Creating Equations**
**A-REI: Algebra- Reasoning with Equations and Inequalities**
**F-IF:  Functions – Interpreting Functions**
**F-BF: Functions – Building Functions**
**F-TF: Functions - Trigonometric Functions**

**Unit Overview**

Students will build the definition of the derivative using the concept of limits and use the derivative to compute the instantaneous rate of change of a function.  Students will learn the rules of finding derivatives of functions such as the constant multiple rule, product rule, quotient rule, and chain rule.  They will also learn how to find the derivative of polynomials, trigonometric functions, logarithmic and exponential functions.  They will see the connection between the derivative function and the slope of a curve.  The concept of the derivative will be applied to linear motion problems and their application to physics.

**Standard Number(s)     * i.e:   Math: F.LE.A.4       i.e.: NJSLSA.R4.**

N-RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*

N-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-RN.B.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

N-Q.A.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays

A-SSE.A.1. Interpret expressions that represent a quantity in terms of its context

A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it

A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features* *include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-BF.A.1 Write a function that describes a relationship between two quantities.

F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF.B.4 Find inverse functions.

F-BF.B.5 Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

F-TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.A.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for πx, π+x, and 2π–x in terms of their values for x, where x is any real number.

F-TF.A.4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.

9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

K-12.MP.1 Make sense of problems and persevere in solving them.

K-12.MP.2 Reason abstractly and quantitatively.

K-12.MP.3 Construct viable arguments and critique the reasoning of others.

K-12.MP.4 Model with mathematics

K-12.MP.5 Use appropriate tools strategically.

K-12.MP.6 Attend to precision

K-12.MP.7 Look for and make use of structure.

K-12.MP.8 Look for and express regularity in repeated reasoning.

**Intended Outcomes - {Essential Questions}**

- What is the definition of the derivative of a function?

- What is the definition of a derivative of a function at a point?
- How does the derivative function relate to the slope of a curve?
- What are the definitions of left-handed and right-handed derivatives?
- What conditions must be must in order for a derivative to exist?
- What are some common forms of notation of a derivative?
- What does a derivative represent graphically?
- How can you approximate a graph of f'(x) given a table of values for f(x)?

**Enduring Understandings**

- Functions can be analyzed graphically by their limiting behavior and rates of change.
- Functions can be analyzed using their table of values.
- Technology can be used at various stages to enhance understanding using its power to visualize and computate
- The relationship between a derivative function and the slope of a curve or the instantaneous rate of change of a function
- The relationship between position of an object and its velocity and acceleration as derivative functions
- What it means for the derivative to exist at a point
- How to apply the derivative rules to calculate derivatives of various functions

| *Check all that apply.* **21st Century Themes** | | *Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills** | |
|---|---|---|---|
| | **Global Awareness** | X | **Creativity and Innovation** |
| | **Environmental Literacy** | X | **Critical Thinking and Problem Solving** |
| | **Health Literacy** | X | **Communication** |
| | **Civic Literacy** | X | **Collaboration** |
| X | **Financial, Economic, Business, and Entrepreneurial Literacy** | | |

**Student Learning Targets/Objectives (Students will know/Students will understand)**

- calculate slopes and derivatives using the definition of the derivative
- graph a function from the graph of its derivative, graph the derivative of a function from its graph
- find where a function is not differentiable and distinguish between corners, cusps, discontinuities, and vertical tangents
- approximate derivatives numerically and graphically

- use the rules of differentiation to calculate derivatives of polynomial, rational, exponential, logarithmic, and trigonometric functions
- use derivatives to analyze straight line motion and solve other problems involving rates of change
- differentiate composite functions using the Chain Rule
- find the derivative of an implicitly defined curve using implicit differentiation
- to find the derivative of inverse functions
- calculate higher order derivatives (i.e. second and third derivatives)
- calculate derivatives of functions involving the inverse trigonometric functions
- write the equation of a line tangent to a curve
- write the equation of line normal to a curve
- calculate the derivative at a point on the graphing calculator

**Find an equation for the line tangent to the curve at the given value of $x$.**

- $y = x^4 - 3x^2 + 2$ at $x = 1$
- $y = \frac{x-1}{x+1}$ at $x = 2$
- $y = x \sin x + \cos x$ at $x = \pi$

| $x$ | 3.3 | 3.4 | 3.5 | 3.6 | 3.7 |
|---|---|---|---|---|---|
| $f(x)$ | 3.69 | 3.96 | 4.25 | 4.56 | 4.89 |

Let $f$ be a differentiable function that is defined for all real numbers $x$. Use the table above to estimate $f'(3.6)$.

(A) 0.3 (B) 1.8 (C) 2.7 (D) 3.0 (E) 3.2

Given $f(x) = x^3$, calculate:

1. $f(1.999) =$
2. $f(2) =$
3. $f(2.001) =$
4. Estimate $f'(2)$ using the symmetric difference quotient.
5. Compare your answer in #4 with $nDeriv(x^3, x, 2)$. How do they compare?
6. What is the $nDeriv($ giving you?
7. To graph a derivative, go to $Y=$ and enter $y1 = nDeriv(f(x), x, x)$.

Given the following table of values, find the indicated derivatives in parts (a) and (b).

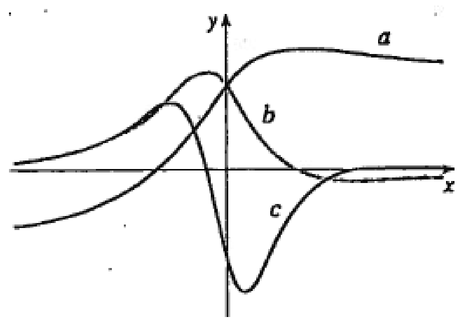
| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 2 | 1 | 7 |
| 8 | 5 | −3 |

(a) $g'(2)$, where $g(x) = [f(x)]^3$

If $y^2 - 3x = 7$, then $\frac{d^2y}{dx^2} =$

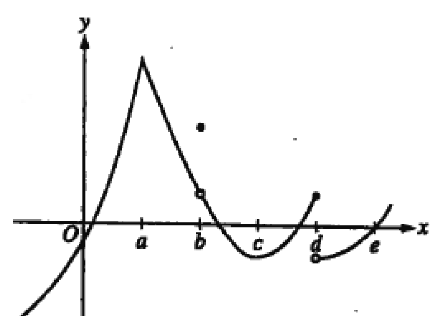
(a) $-\frac{6}{7y^3}$ (b) $-\frac{3}{y^3}$ (c) 3 (d) $\frac{3}{2y}$ (e) $-\frac{9}{4y^3}$

The figure shows the graphs of three functions – ***position of a car, velocity of a car, acceleration of a car***. Which is which? Why?




The graph of a function $f$ is shown. At which value of $x$ is $f$ continuous, but not differentiable?

(a) $a$ (b) $b$ (c) $c$ (d) $d$ (e) $e$




Graph of $f$

$\lim_{h\to 0}\left(\frac{\tan\left(\frac{\pi}{4}+h\right)-\tan\left(\frac{\pi}{4}\right)}{h}\right) =$

(a) $-\sqrt{2}$ (b) $-2$ (c) 2 (d) $\sqrt{2}$ (e) nonexistent

| |
|---|
| **Assessments (Pre, Formative, Summative, Other)** ***Denote required common assessments with an **** |
| Quizzes<br>Benchmark Tests: Midterm/Final Exam<br>Homework<br>Unit Test<br>Sample AP Multiple Choice problems<br>Warm up problems |

| **Teaching and Learning Activities** | |
|---|---|
| *Activities* | The course lends itself to a mode of instruction that engages the students in a multi-representational approach to studying calculus concepts. Many "what if' questions provide great insights or a basis for discussion and help to validate understanding at various levels of development as each unit is explored. While problem sets and explorations may be rigorous and challenging, they are designed so that the students can be successful if they utilize those resources available to them such as classroom discussions, group explorations, projects, and peer and instructor interactions. Instructional modes attempt to address the variety of learning styles that include visual learners, auditory learners, and kinesthetic learners.<br><br>Students use Desmos to create a slider showing a function and its derivative. The graph shows the tangent line sliding along the curve as its slope values are plotted. Polynomial, rational, exponential, logarithmic, trigonometric, and inverse trigonometric functions are used as the parent function. |
| *Differentiation Strategies* | Alternative assessment projects to demonstrate mastery<br>Peer to Peer Tutoring<br>Implementation of Visual Representations- Calculus in Motion<br>Technology Implementation<br>Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of limits<br>Allow students to work in small groups<br>Provide access to Khan Academy Videos<br>Provide opportunities for questions<br>Differentiation Strategies for Special Education Students<br>Differentiation Strategies for Gifted and Talented Students<br>Differentiation Strategies for ELL Students<br>Differentiation Strategies for At Risk Students |
| *Honors* | Completing AP free-response questions developed by College Board |

| Resources |
|---|
| www.collegeboard.com: provides information about AP exams<br><br>http://archives.math.utk.edu/visual.calculus/: an extremely thorough site for pre-calculus through integral calculus with accompanying tutorials, practice problems, interactive quizzes and animations<br><br>http://online.math.uh.edu/HoustonACT/: contains Powerpoint presentations<br><br>www.calculus-help.com/funstuff/phobe.html: animated explanations of the first two chapters of calculus<br><br>www.calculusabc.com: teacher and student resources containing multiple choice problems for each unit and well as a forum for teachers to exchange teaching ideas<br><br>http://clem.mscd.edu/~talmanl/APCalculus.html: FRQ solutions, explanations, links to number of other sites and much more<br><br>http://www.calculus.org/<br><br>http://www.calculus-help.com/<br><br>AP Central<br><br>Khan Academy<br><br>Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006. |

| | |
|---|---|
| **Content Area/ Grade Level/ Course:** | **Mathematics**<br>**12**<br>**Calculus Honors** |
| **Unit Plan Title:** | **Applications of Derivatives** |
| **Time Frame** | **30 days** |

| Anchor Standards/Domain*     *i.e: ELA: reading, writing i.e.: Math: Algebra |
|---|
| **N-RN: Number and Quantity – The Real Number System**<br>**N-Q: Number and Quantity – Quantity**<br>**A-SSE: Algebra – Seeing Structure in Expressions**<br>**A-APR: Algebra – Arithmetic with Polynomials and Rational Expressions**<br>**A-CED: Algebra – Creating Equations** |

**A-REI: Algebra- Reasoning with Equations and Inequalities**
**F-IF: Functions – Interpreting Functions**
**F-BF: Functions – Building Functions**
**F-TF: Functions - Trigonometric Functions**
**G-SRT: Geometry- Similarity, Right Triangles, and Trigonometry**
**G-GMD: Geometry: Geometric Measurement and Dimension**
**G-MG: Modeling with Geometry**

## Unit Overview

Students will now be able to apply their knowledge of derivatives and derivative rules to solve real world problems. They will be able to analyze a function (i.e. find intervals of increasing/decreasing, intervals of concavity, extrema) based on information about its first and second derivatives. The idea of locating maximums and minimums will extend to setting up and solving optimization problems. Students will also apply the concept of rates of change to solve related rate problems. Also, they will implement their knowledge of the tangent line to approximate function values through linearization. Students will solve optimization problems. Applications to physics, business, and geometry will be investigated.

## Standard Number(s) * i.e: Math: F.LE.A.4 i.e.: NJSLSA.R4.

N-RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

N-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-RN.B.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

N-Q.A.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays

A-SSE.A.1. Interpret expressions that represent a quantity in terms of its context

A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it

A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-BF.A.1 Write a function that describes a relationship between two quantities.

F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF.B.4 Find inverse functions.

F-BF.B.5 Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

F-TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.A.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi x$, $\pi+x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

F-TF.A.4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

G-SRT.B.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-SRT.C.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

G-MG.A.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
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K-12.MP.5 Use appropriate tools strategically.

K-12.MP.6 Attend to precision

K-12.MP.7 Look for and make use of structure.

K-12.MP.8 Look for and express regularity in repeated reasoning.

**Intended Outcomes - {Essential Questions}**

- What information can be found from the graph of the first derivative to help sketch the graph of $f(x)$?
- How are derivative functions used in problem solving when modeling real world situations?
- What does it mean to be “locally linear”?
- What is the connection between first and second derivative tests in the application process?

**Enduring Understandings**

- Functions can be analyzed graphically by their limiting behavior and rates of change.
- Functions can be analyzed using their table of values.
- Technology can be used at various stages to enhance understanding using its power to visualize and computate
- A function’s first derivative can be used to analyze the behavior of a function
- The derivative has multiple interpretations and applications to optimization and rates of change
- Tangent lines can be used as approximating functions

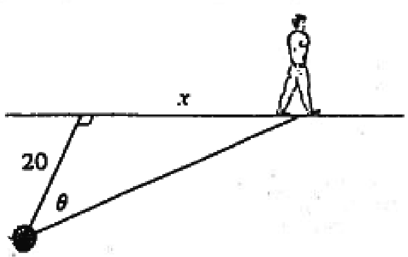
| | *Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills** |
|---|---|

| | | | |
|---|---|---|---|
| | **Global Awareness** | X | **Creativity and Innovation** |
| | **Environmental Literacy** | X | **Critical Thinking and Problem Solving** |
| | **Health Literacy** | X | **Communication** |
| | **Civic Literacy** | X | **Collaboration** |
| X | **Financial, Economic, Business, and Entrepreneurial Literacy** | | |

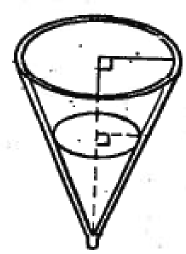
**Student Learning Targets/Objectives (Students will know/Students will understand)**

- locate critical values of a function using derivatives
- apply the Mean Value Theorem and Extreme Value Theorem
- use the first and second derivative tests to determine local and absolute extreme values of a function
- determine concavity of a function and locate points of inflection by analyzing second derivatives
- graph a function using first and second derivative information
- solve application problems involving finding minimum or maximum values of a function
- find linearizations and apply the result when approximating functional values
- calculate the error in approximation when using a tangent-line approximation
- solve related rate problems
- apply L'Hopital's rule to evaluate limits with indeterminate forms

A man walks along a path at a speed of 4 feet per second. A searchlight is located on the ground 20 feet from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 feet from the point on the path closest to the searchlight?




A conical funnel is 14 inches in diameter and 12 inches deep. Water is flowing out at a rate of 40 cubic inches per second. How fast is the depth of the liquid falling when the level is 6 inches deep?



The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2,4]$ occurs at $x =$

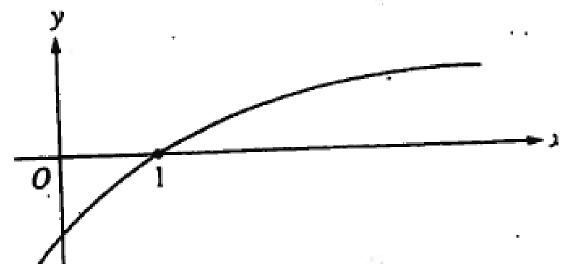
(a) 4 (b) 2 (c) 1 (d) 0 (e) $-2$

Let $f$ be the function defined by $f(x) = x + \ln x$. What is the value of $c$ for which the instantaneous rate of change of $f$ at $x = c$ is the same as the average rate of change of $f$ over the interval [1,3]?

(a) 1.820 (b) 1.944 (c) 2.164 (d) 2.342 (e) 2.452

The graph of a twice-differentiable function $f$ is shown. Which of the following is true?

(a) $f(1) < f'(1) < f''(1)$ (b) $f(1) < f''(1) < f'(1)$
(c) $f'(1) < f(1) < f''(1)$ (d) $f''(1) < f(1) < f'(1)$
(e) $f''(1) < f'(1) < f(1)$



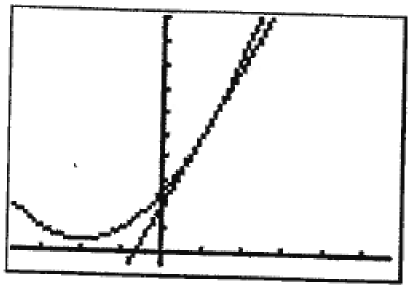

The position of an object attached to a spring is given by $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$, where $t$ is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

(a) Zero (b) Three (c) Five (d) Six (e) Seven

A manufacturer wants to paint a closed container having a square base with rectangular sides. Each container must have the capacity to hold 96 cubic meters. The paint needed for the top only costs \$0.06 per square meter. The paint needed for the bottom and the sides of the container costs \$0.03 per square meter. What dimensions of the container will minimize the total cost of painting this container?

What are the dimensions of the lightest open-top right circular cylindrical can that holds a volume of $1000\text{cm}^3$

- Estimate $f(1.2)$ using the line tangent to the graph of $f(x) = x^2 + 4x + 5$ at $x = 1$.
- Is this an overestimate or underestimate of $f(1.2)$?



Over what interval(s) is $f$ increasing?

Over what interval(s) is $f$ decreasing?

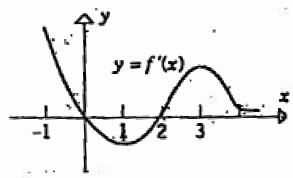
At what value(s) of $x$ does $f$ have a local maxima?

At what value(s) of $x$ does $f$ have a local minima?

Over what interval(s) of $x$ is $f$ concave upward?

Over what interval(s) of $x$ is $f$ concave downward?

At what value(s) of $x$ does $f$ have an inflection point?




Given $\frac{dy}{dx} = 1 - y$. Let y = f(x). If f (1) = 0, find the $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$

| Assessments (Pre, Formative, Summative, Other) *Denote required common assessments with an ** |
|---|
| Quizzes<br>Benchmark Tests: Midterm/Final Exam<br>Homework<br>Unit Test<br>Sample AP Multiple Choice problems<br>Warm up problems |

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|---|---|
| *Activities* | The course lends itself to a mode of instruction that engages the students in a multi-representational approach to studying calculus concepts. Many "what if' questions provide great insights or a basis for discussion and help to validate understanding at various levels of development as each unit is explored. While |

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|---|---|
| | problem sets and explorations may be rigorous and challenging, they are designed so that the students can be successful if they utilize those resources available to them such as classroom discussions, group explorations, projects, and peer and instructor interactions. Instructional modes attempt to address the variety of learning styles that include visual learners, auditory learners, and kinesthetic learners.<br><br>Building Geometric Designs that optimize values<br><br>Class Matching Game of f to f' |
| *Differentiation Strategies* | Alternative assessment projects to demonstrate mastery<br>Peer to Peer Tutoring<br>Implementation of Visual Representations- Calculus in Motion<br>Technology Implementation<br>Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of limits<br>Allow students to work in small groups<br>Provide access to Khan Academy Videos<br>Provide opportunities for questions<br>Differentiation Strategies for Special Education Students<br>Differentiation Strategies for Gifted and Talented Students<br>Differentiation Strategies for ELL Students<br>Differentiation Strategies for At Risk Students |
| *Honors* | Completing AP free-response questions developed by College Board |
| **Resources** | |

www.collegeboard.com: provides information about AP exams

http://archives.math.utk.edu/visual.calculus/: an extremely thorough site for pre-calculus through integral calculus with accompanying tutorials, practice problems, interactive quizzes and animations

http://online.math.uh.edu/HoustonACT/: contains Powerpoint presentations

www.calculus-help.com/funstuff/phobe.html: animated explanations of the first two chapters of calculus

www.calculusabc.com: teacher and student resources containing multiple choice problems for each unit and well as a forum for teachers to exchange teaching ideas

http://clem.mscd.edu/~talmanl/APCalculus.html: FRQ solutions, explanations, links to number of other sites and much more

http://www.calculus.org/

http://www.calculus-help.com/

AP Central

Khan Academy

Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.

| | |
|---|---|
| **Content Area/ Grade Level/ Course:** | **Mathematics**<br>**12**<br>**Calculus Honors** |
| **Unit Plan Title:** | **The Definite Integral** |
| **Time Frame** | **30 days** |

**Anchor Standards/Domain*      *i.e: ELA: reading, writing  i.e.:  Math: Algebra**

**N-RN: Number and Quantity – The Real Number System**
**N-Q: Number and Quantity – Quantity**
**A-SSE: Algebra – Seeing Structure in Expressions**
**A-APR: Algebra – Arithmetic with Polynomials and Rational Expressions**
**A-CED: Algebra – Creating Equations**
**A-REI: Algebra- Reasoning with Equations and Inequalities**
**F-IF:  Functions – Interpreting Functions**
**F-BF: Functions – Building Functions**
**F-TF: Functions - Trigonometric Functions**
**G-SRT: Geometry- Similarity, Right Triangles, and Trigonometry**
**G-GMD: Geometry: Geometric Measurement and Dimension**
**G-MG: Modeling with Geometry**

**Unit Overview**

Students will learn the definition of the definite integral as a Riemann sum and be able to approximate the definite integral using various methods. They will be able to use approximating methods such as the Rectangular Approximating Method and the Trapezoid Rule. They will also learn to use geometry formulas to calculate the signed area between the graph of $f(x)$ and the $x$-axis. Students will also familiarize themselves with basic techniques of integration and the properties of the definite integral. The definite integral will be thought of as an accumulating function and students will be able to evaluate integrals by the Fundamental Theorem of Calculus. Students will also apply the Fundamental Theorem of Calculus to see the connection between integration and differentiation of functions.

**Standard Number(s)     * i.e:   Math: F.LE.A.4       i.e.: NJSLSA.R4.**

N-RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

N-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-RN.B.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

N-Q.A.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays

A-SSE.A.1. Interpret expressions that represent a quantity in terms of its context

A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it

A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-BF.A.1 Write a function that describes a relationship between two quantities.

F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF.B.4 Find inverse functions.

F-BF.B.5 Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

F-TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.A.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi x$, $\pi+x$, and $2\pi-x$ in terms of their values for $x$, where $x$ is any real number.

F-TF.A.4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

G-SRT.B.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-SRT.C.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

G-MG.A.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.

9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

K-12.MP.1 Make sense of problems and persevere in solving them.

K-12.MP.2 Reason abstractly and quantitatively.

K-12.MP.3 Construct viable arguments and critique the reasoning of others.

K-12.MP.4 Model with mathematics

K-12.MP.5 Use appropriate tools strategically.

K-12.MP.6 Attend to precision

K-12.MP.7 Look for and make use of structure.

K-12.MP.8 Look for and express regularity in repeated reasoning.

**Intended Outcomes - {Essential Questions}**

- What are the different ways you can use Riemann Sums to approximate area under a curve and how is that related to integration?
- What is a definite integral?

- How are definite integrals like limits?
- How do you use the fundamental theorem of calculus to evaluate definite integrals and anti- derivatives?
- How do you differentiate functions defined by an integral?

**Enduring Understandings**

- Functions can be analyzed graphically by their limiting behavior and rates of change.
- Functions can be analyzed using their table of values.
- Technology can be used at various stages to enhance understanding using its power to visualize and compute
- Antidifferentiation is the inverse process of differentiation
- The definite integral of a function over an interval is the limit of a Riemann sum
- The definite integral of a function has many interpretations and applications involving accumulation
- The Fundamental Theorem of Calculus can be applied to evaluate definite integrals

*Indicate whether these skills are* ***E****-Encouraged,* ***T****-Taught, or* ***A****-Assessed in this unit by marking* ***E, T, A*** *on the line before the appropriate skill.*

**21st Century Skills**

| | | | |
|---|---|---|---|
| | **Global Awareness** | X | **Creativity and Innovation** |
| | **Environmental Literacy** | X | **Critical Thinking and Problem Solving** |
| | **Health Literacy** | X | **Communication** |
| | **Civic Literacy** | X | **Collaboration** |
| X | **Financial, Economic, Business, and Entrepreneurial Literacy** | | |

**Student Learning Targets/Objectives (Students will know/Students will understand)**

- approximate the area under the graph of a non-negative continuous function using rectangular approximation methods (RAM)
- interpret the area under the graph as a net accumulation of a rate of change
- express the area under a curve as a definite integral and as a limit of Riemann sums and calculate the area
- apply the rules for definite integrals and find the average value of a function over a closed interval (use Mean Value Theorem for Integrals)
- apply and understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus
- define a function using a definite integral and calculate its derivatives
- evaluate definite integrals using the Fundamental Theorem of Calculus
- approximate the definite integral using the Trapezoidal Rule
- construct antiderivatives using the Fundamental Theorem of Calculus

The table below gives the values for the rate, in gallons per second, at which water flowed into a lake, with readings taken at specific times. A rectangular approximation using right –endpoint rectangles (RRAM), with the five subintervals indicated by the data in the table, is used to estimate the total amount of water that flowed into the lake during the time period $0 \le t \le 60$. What is this estimate?

| Time (sec) | 0 | 10 | 25 | 37 | 46 | 60 |
|---|---|---|---|---|---|---|
| Rate (gal/sec) | 500 | 400 | 350 | 280 | 200 | 180 |

(a) 1,910 gal (b) 14,100 gal (c) 16,930 gal (d) 18,725 gal (e) 20,520 gal

Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

| $t$ (days) | $W(t)$ (°C) |
|---|---|
| 0 | 20 |
| 3 | 31 |
| 6 | 28 |
| 9 | 24 |
| 12 | 22 |
| 15 | 21 |

If $f(x)=\begin{cases} x, & 0\le x\le 2 \\ -x+4, & 2<x\le 4 \end{cases}$, then $\int_0^4 f(x)\,dx =$

$$\int_{-1}^{2}\left(3x^2+2x-16\right)dx =$$

$$\int_1^4 \frac{3x-4}{\sqrt{x}}\,dx =$$

$$\int_0^2\left(2x^3-6x+\frac{3}{1+x^2}\right)dx =$$

$$\int_0^3 3^x\,dx =$$

$$\int_2^6 \frac{4}{x}\,dx =$$

$$\int_0^{\frac{\pi}{3}}\left(2x+\sec x\tan x\right)dx =$$

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$$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{4+\frac{5i}{n}}\cdot\frac{5}{n}=?$$

**Choose 1 answer:**

(A) $\int_4^9 \sqrt{4+x}\,dx$

(B) $\int_0^5 \sqrt{x}\,dx$

(C) $\int_4^9 \sqrt{x}\,dx$

(D) $\int_0^4 \sqrt{4+x}\,dx$

| Assessments (Pre, Formative, Summative, Other) | *Denote required common assessments with an ** |
|---|---|
| Quizzes<br>Benchmark Tests: Final Exam<br>Homework<br>Unit Test<br>Sample AP Multiple Choice problems<br>Warm up problems | |

| **Teaching and Learning Activities** | |
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| *Activities* | The course lends itself to a mode of instruction that engages the students in a multi-representational approach to studying calculus concepts. Many "what if' questions provide great insights or a basis for discussion and help to validate understanding at various levels of development as each unit is explored. While problem sets and explorations may be rigorous and challenging, they are designed so that the students can be successful if they utilize those resources available to them such as classroom discussions, group explorations, projects, and peer and instructor interactions. Instructional modes attempt to address the variety of learning styles that include visual learners, auditory learners, and kinesthetic learners.<br><br>Students write an expression for an approximation of the area between the x axis and the graph of $f(x)$ for a particular function given as a formula on a specified interval as a left, right, and midpoint Riemann sum using $n$ subdivisions. They then use a Desmos graph with slider to explore sums. The file superimposes rectangular areas on the graph of $f(x)$, showing the sum value. Also, students will write limits of their Riemann suns as $n$ goes to infinity, then identify each as a definite integral, and use the Fundamental Theorem of Calculus to evaluate the integral.<br><br>Matching Index Card Activity from Integral to Riemann Sum<br><br>Classwide Calculus In Motion Mean Value Theorem Guessing Game |
| *Differentiation Strategies* | Alternative assessment projects to demonstrate mastery<br>Peer to Peer Tutoring<br>Implementation of Visual Representations- Calculus in Motion<br>Technology Implementation<br>Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of limits<br>Allow students to work in small groups<br>Provide access to Khan Academy Videos<br>Provide opportunities for questions<br>Differentiation Strategies for Special Education Students<br>Differentiation Strategies for Gifted and Talented Students<br>Differentiation Strategies for ELL Students<br>Differentiation Strategies for At Risk Students |
| *Honors* | Completing AP free-response questions developed by College Board |

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| Resources |
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| www.collegeboard.com: provides information about AP exams<br><br>http://archives.math.utk.edu/visual.calculus/: an extremely thorough site for pre-calculus through integral calculus with accompanying tutorials, practice problems, interactive quizzes and animations<br><br>http://online.math.uh.edu/HoustonACT/: contains Powerpoint presentations<br><br>www.calculus-help.com/funstuff/phobe.html: animated explanations of the first two chapters of calculus<br><br>www.calculusabc.com: teacher and student resources containing multiple choice problems for each unit and well as a forum for teachers to exchange teaching ideas<br><br>http://clem.mscd.edu/~talmanl/APCalculus.html: FRQ solutions, explanations, links to number of other sites and much more<br><br>http://www.calculus.org/<br><br>http://www.calculus-help.com/<br><br>AP Central<br><br>Khan Academy<br><br>Texas Instruments for AP<br><br>Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006. |

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| **Content Area/ Grade Level/ Course:** | **Mathematics**<br>**12**<br>**Calculus Honors** |
| **Unit Plan Title:** | **Applcations of the Defninte Integral and Differential Equations** |
| **Time Frame** | **45 days** |

**Anchor Standards/Domain*      *i.e: ELA: reading, writing  i.e.:  Math: Algebra**

**N-RN: Number and Quantity – The Real Number System**
**N-Q: Number and Quantity – Quantity**
**A-SSE: Algebra – Seeing Structure in Expressions**
**A-APR: Algebra – Arithmetic with Polynomials and Rational Expressions**
**A-CED: Algebra – Creating Equations**
**A-REI: Algebra- Reasoning with Equations and Inequalities**
**F-IF:  Functions – Interpreting Functions**
**F-BF: Functions – Building Functions**
**F-TF: Functions - Trigonometric Functions**
**G-SRT: Geometry- Similarity, Right Triangles, and Trigonometry**
**G-GMD: Geometry: Geometric Measurement and Dimension**
**G-MG: Modeling with Geometry**

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| **Unit Overview** |
| Once students are familiar with the definite integral and antidifferention techniques, they will explore applications to finding areas in the plane and volumes of solids. They will revisit one dimensional motion and how definite integrals can calculate displacement and distance traveled. Students will also discover how to solve differential equations graphically through slope fields and algebraically through initial value problems. Students will learn more advanced tools of integration such as u-substitution, integration by parts, and partial fractions. |
| **Standard Number(s) * i.e: Math: F.LE.A.4 i.e.: NJSLSA.R4.** |
| N-RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*<br><br>N-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.<br><br>N-RN.B.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.<br><br>N-Q.A.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays<br><br>A-SSE.A.1. Interpret expressions that represent a quantity in terms of its context<br><br>A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it<br><br>A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression<br><br>A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.<br><br>A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.<br><br>A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*<br><br>A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.<br><br>A-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*<br><br>A-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations<br><br>A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. |

F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-BF.A.1 Write a function that describes a relationship between two quantities.

F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF.B.4 Find inverse functions.

F-BF.B.5 Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

F-TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.A.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi x$, $\pi+x$, and $2\pi–x$ in terms of their values for x, where x is any real number.

F-TF.A.4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

G-SRT.B.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-SRT.C.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

G-MG.A.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

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8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

K-12.MP.1 Make sense of problems and persevere in solving them.

K-12.MP.2 Reason abstractly and quantitatively.

K-12.MP.3 Construct viable arguments and critique the reasoning of others.

K-12.MP.4 Model with mathematics

K-12.MP.5 Use appropriate tools strategically.

K-12.MP.6 Attend to precision

K-12.MP.7 Look for and make use of structure.

K-12.MP.8 Look for and express regularity in repeated reasoning.

**Intended Outcomes - {Essential Questions}**

- How does a slope field help approximate a function?
- How do you solve a differential equation?
- When can u-substitution be applied and how can we use it to integrate?
- How are differential equations used to solve real-world problems?
- How can a definite integral calculate the area and volumes of uncommon shapes and solids?

**Enduring Understandings**

- Functions can be analyzed graphically by their limiting behavior and rates of change.
- Functions can be analyzed using their table of values.
- Technology can be used at various stages to enhance understanding using its power to visualize and compute
- The solution to a differential equation can be represented graphically through a slope field or algebraically through antiderivatives
- The definite integral can be applied to motion problems to calculate displacement and distance traveled
- Separation of variables is required when solving a differential equation in terms of $x$ and $y$
- Volume can be calculated by integrating the area of cross-sections over an interval

| | | | |
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| | | *Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*<br>**21st Century Skills** | |
| | **Global Awareness** | X | **Creativity and Innovation** |
| | **Environmental Literacy** | X | **Critical Thinking and Problem Solving** |
| | **Health Literacy** | X | **Communication** |

| | | | |
|---|---|---|---|
| ☐ | Civic Literacy | X | Collaboration |
| X | Financial, Economic, Business, and Entrepreneurial Literacy | | |

**Student Learning Targets/Objectives (Students will know/Students will understand)**

- solve initial value problems given a derivative and initial condition
- construct slope fields using technology and interpret slope fields as visualizations of differential equations
- solve differential equations of the form $dy/dx = f(x)$
- compute indefinite and definite integrals by the method of substitution
- compute indefinite and definite integrals by the method of integration by parts
- solve differential equations of the form $dy/dx = f(x) \cdot g(y)$ in which the variables are separable
- solve problems involving exponential growth in a variety of applications
- to solve problems in which a rate is integrated to find the net change over time in a variety of applications
- use a definite integral to calculate total distance traveled by a particle over an interval
- use integration to calculate areas of regions in a plane
- use integration by slicing to calculate volumes of solids with known cross sectional areas or formed by rotation of a bounded region about a given axis
- find the average value of a function using the Mean Value Theorem

Let $f(x) = \int_0^{x^2} \sqrt{\sin t}\, dt$. What is the value of $f'(1)$?

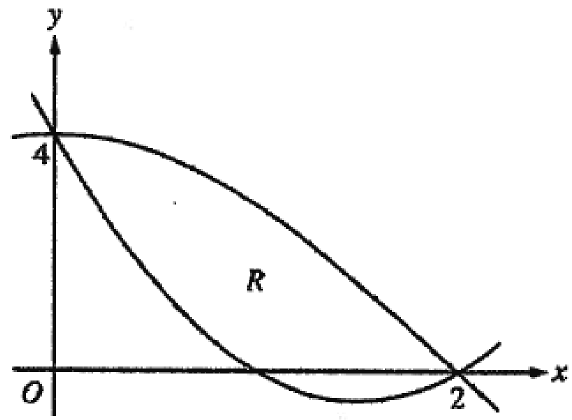
a. 0 b. 1.132 c. 1.264 d. 0.917 e. 1.835

A pizza, heated to a temperature of 350°F, is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at the rate of $-110e^{-0.4t}$ °F per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(a) 112°F (b) 119°F (c) 147°F (d) 238°F (e) 335°F

If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(a) $e^{\tan x} + 4$ (b) $e^{\tan x} + 5$ (c) $5e^{\tan x}$ (d) $\tan x + 5$ (e) $\tan x + 5e^x$

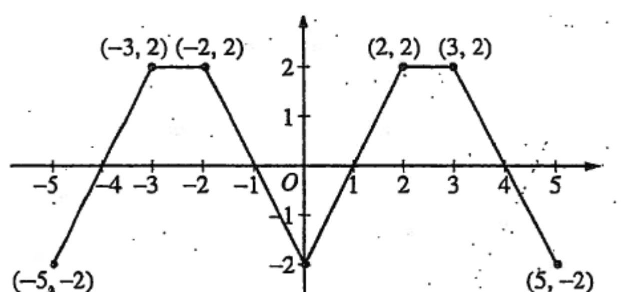



Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let $R$ be the region bounded by the graphs of $f$ and $g$, as shown in the figure above.

(a) Find the area of $R$.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 4$.

(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.




Graph of $f$

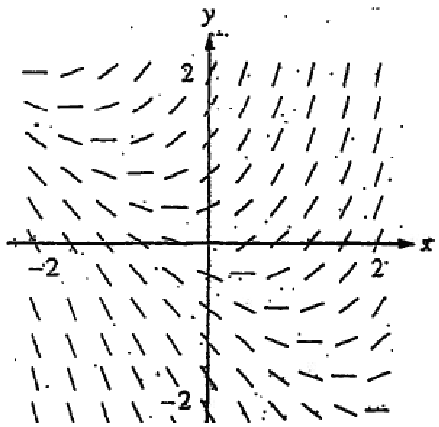
The graph of the function $f$ shown above consists of six line segments. Let $g$ be the function given by $g(x) = \int_0^x f(t)\, dt$.

Suppose that $f$ is defined for all real numbers $x$ and is periodic with a period of length 5. The graph above shows two periods of $f$. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of $g$ at $x = 108$.

Shown is the slope field for which of the following differential equations?

(a) $\frac{dy}{dx}=1+x$ (b) $\frac{dy}{dx}=x^2$

(c) $\frac{dy}{dx}=x+y$ (d) $\frac{dy}{dx}=\frac{x}{y}$ (e) $\frac{dy}{dx}=\ln y$




**Assessments (Pre, Formative, Summative, Other)** ***Denote required common assessments with an ****

Quizzes
Benchmark Tests: Final Exam
Homework
Unit Test
Sample AP Multiple Choice problems
Warm up problems

**Teaching and Learning Activities**

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| *Activities* | The course lends itself to a mode of instruction that engages the students in a multi-representational approach to studying calculus concepts. Many "what if' questions provide great insights or a basis for discussion and help to validate understanding at various levels of development as each unit is explored. While problem sets and explorations may be rigorous and challenging, they are designed so that the students can be successful if they utilize those resources available to them such as classroom discussions, group explorations, projects, and peer and instructor interactions. Instructional modes attempt to address the variety of learning styles that include visual learners, auditory learners, and kinesthetic learners.<br><br>To visualize the steps necessary to find the volume of a solid with known cross sections, students build a physical model with foam board of weighted paper to construct several cross sections. |
| *Differentiation Strategies* | Alternative assessment projects to demonstrate mastery<br>Peer to Peer Tutoring<br>Implementation of Visual Representations- Calculus in Motion<br>Technology Implementation<br>Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of limits<br>Allow students to work in small groups<br>Provide access to Khan Academy Videos<br>Provide opportunities for questions<br>Differentiation Strategies for Special Education Students<br>Differentiation Strategies for Gifted and Talented Students<br>Differentiation Strategies for ELL Students |

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| | Differentiation Strategies for At Risk Students |
| *Honors* | Completing AP free-response questions developed by College Board |

**Resources**

www.collegeboard.com: provides information about AP exams

http://archives.math.utk.edu/visual.calculus/: an extremely thorough site for pre-calculus through integral calculus with accompanying tutorials, practice problems, interactive quizzes and animations

http://online.math.uh.edu/HoustonACT/: contains Powerpoint presentations

www.calculus-help.com/funstuff/phobe.html: animated explanations of the first two chapters of calculus

www.calculusabc.com: teacher and student resources containing multiple choice problems for each unit and well as a forum for teachers to exchange teaching ideas

http://clem.mscd.edu/~talmanl/APCalculus.html: FRQ solutions, explanations, links to number of other sites and much more

http://www.calculus-help.com/

AP Central

Khan Academy
Texas Instruments for AP

Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.